FM Geometry Name: Key

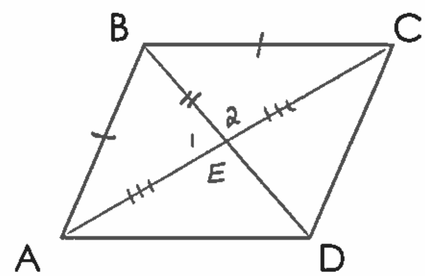
5-4 Special Parallelograms Date: ______________ Pd: ____

Theorem: **A parallelogram is a rhombus if and only if its diagonals are perpendicular.**

1. If a parallelogram is a rhombus, then its diagonals are perpendicular.

Given: Rhombus ABCD

Prove: $\overline{AC} \perp \overline{BD}$

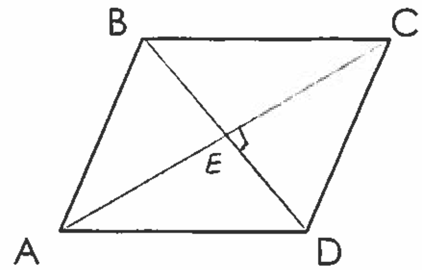


Statements	Reasons
1. ABCD is a Rhombus	1. Given
2. $\overline{AB} \cong \overline{BC}$	2. Def. of rhombus
3. $\overline{BE} \cong \overline{BE}$	3. Refl. Prop. of $\cong$
4. E is the midpoint of $\overline{AC}$	4. Diagonals of a ▱ bisect each other
5. $\overline{AE} \cong \overline{EC}$	5. Def. of Midpoint
6. $\triangle ABE \cong \triangle CBE$	6. SSS $\cong$ Post.
7. $\angle 1 \cong \angle 2$	7. CPCTC
8. $\overline{AC} \perp \overline{BD}$	8. Lines form $\cong$ adj. $\angle$s $\rightarrow \perp$ lines

2. If the diagonals of parallelogram are perpendicular, then it is a rhombus.

Given: Parallelogram ABCD
$\overline{AC} \perp \overline{BD}$

Prove: ABCD is a Rhombus



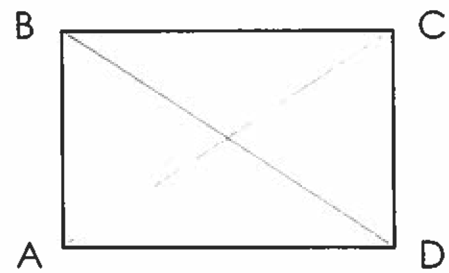
Statements	Reasons
① $\square ABCD$, $\overline{AC} \perp \overline{BD}$	① Given
② $\overline{AC}$ and $\overline{BD}$ bisect each other	② Diagonals of a $\square$ bisect each other
③ E is the midpt of $\overline{AC}$	③ Def. of seg. bisector
④ $\overline{BD}$ is the $\perp$ Bisector of $\overline{AC}$	④ Def. of $\perp$ bisector
⑤ $AB = AD$	⑤ $\perp$ bisector Thrm
⑥ $\overline{AB} \cong \overline{AD}$	⑥ Def. of $\cong$ seg.
⑦ $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$	⑦ Opp. sides of a $\square$ are $\cong$
⑧ $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$	⑧ Trans. Prop. of $\cong$
⑨ ABCD is a rhombus	⑨ Def. of Rhombus

Theorem: **A parallelogram is a rectangle if and only if its diagonals are congruent.**

1. If a parallelogram is a rectangle, then its diagonals are congruent.

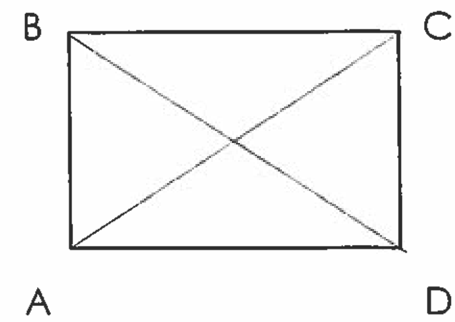
Given: Rectangle ABCD

Prove: $\overline{AC} \cong \overline{DB}$



Statements	Reasons
(1) Rectangle ABCD	(1) Given
(2) $\angle BAD$ and $\angle CDA$ are rt. $\angle$s	(2) Def. of Rectangle
(3) $\angle BAD \cong \angle CDA$	(3) Rt. $\angle$s Thrm
(4) $\overline{AB} \cong \overline{CD}$	(4) Opp. sides of a ▱ are $\cong$
(5) $\overline{AD} \cong \overline{AD}$	(5) Refl. Prop. of $\cong$
(6) $\triangle ABD \cong \triangle DCA$	(6) SAS $\cong$ Post
(7) $\overline{AC} \cong \overline{DB}$	(7) CPCTC

2. If the diagonals of a parallelogram are congruent, then it is a rectangle.



Given: Parallelogram ABCD
$\overline{AC} \cong \overline{BD}$

Prove: ABCD is a rectangle

Statements	Reasons
1. ▱ABCD, $\overline{AC} \cong \overline{BD}$	1. Given
2. $\overline{AD} \cong \overline{AD}$	2. Refl. Prop. of ≅
3. $\overline{AB} \cong \overline{CD}$	3. opp. sides of a ▱ are ≅
4. $\triangle ABD \cong \triangle DCA$	4. SSS ≅ Post
5. $\angle BAD \cong \angle CDA$, $m\angle BAD = m\angle CDA$	5. CPCTC, Def. of ≅ ∠s
6. $\angle BAD$ is supp. to $\angle CDA$	6. Consec. ∠s in a ▱ are supp.
7. $m\angle BAD + m\angle CDA = 180^\circ$	7. Def. of Supp. ∠s
8. $2m\angle BAD = 180^\circ$	8. Subst. Prop. of = (5→7), Dist. Prop.
9. $m\angle BAD = 90^\circ$, $\angle BAD$ is a right ∠	9. Division Prop. of =, Def. of Rt. ∠
10. ABCD is a Rectangle	10. If an angle of a ▱ is a rt. ∠, then it is a rectangle.

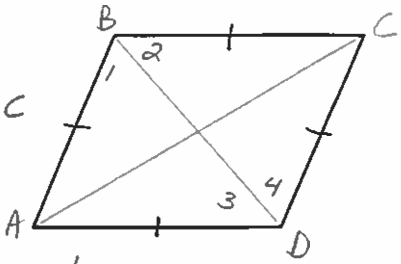
[See Proof on last page!]

Write 2-column proofs for the following Theorems.

Theorem: **Each diagonal of a rhombus bisects two angles of the rhombus.**

Given: Rhombus $ABCD$, Diagonals $\overline{BD}$ and $\overline{AC}$

Prove: $\overline{BD}$ bisects $\angle B$ and $\angle D$, $\overline{AC}$ bisects $\angle A$ and $\angle C$



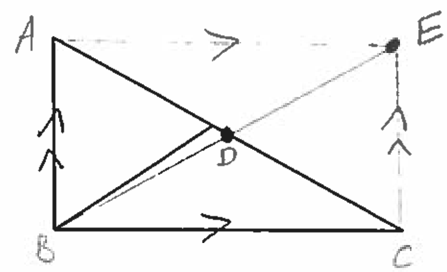
Statements	Reasons
1. Rhombus $ABCD$, Diagonals $\overline{BD}$ and $\overline{AC}$	1. Given
2. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$	2. Def. of rhombus
3. $\overline{BD} \cong \overline{BD}$	3. Refl. Prop. of $\cong$
4. $\triangle ABD \cong \triangle CBD$	4. SSS $\cong$ Post
5. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	5. CPCTC
6. $\overline{BD}$ bisects $\angle B$ and $\angle D$	6. Def. of $\angle$ bisector

Likewise, $\overline{AC}$ bisects $\angle A$ and $\angle C$.

Theorem: **The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.**

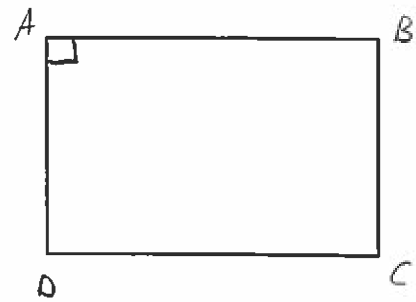
Given: $\angle ABC$ is a Rt $\angle$ and D is the midpoint of $\overline{AC}$

Prove: $DA = DB = DC$



Statements	Reasons
1. $\angle ABC$ is a Right $\angle$, D is the midpt of $\overline{AC}$	1. Given
2. Draw $\overline{AE} \parallel \overline{BC}$ and $\overline{CE} \parallel \overline{AB}$	2. $\parallel$ Post.
3. $ABCE$ is a $\square$	3. Def. of $\square$
4. $ABCE$ is a rectangle	4. $\square$ with 1 right $\angle \rightarrow$ Rectangle [See next Page]
5. $\overline{AC} \cong \overline{BE}$, $AC = BE$	5. Diagonals of a Rectangle are $\cong$, Def. of $\cong$ seg.
6. D is the midpoint of $\overline{BE}$	6. Diagonals of a $\square$ bisect each other
7. $DA = DC = \frac{1}{2}AC$, $DB = \frac{1}{2}BE$	7. Midpoint Thrm
8. $DB = \frac{1}{2}AC$	8. Subst. Prop. of $=$ $(5 \rightarrow 7)$
9. $DA = DB = DC$	9. Trans. Prop. of $=$

Theorem: **If an angle of a parallelogram is a right angle, then the parallelogram is a rectangle.**

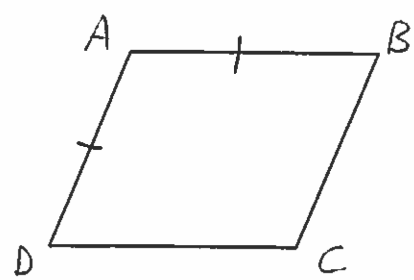


Given: $\square ABCD$ with rt $\angle A$

Prove: ABCD is a rectangle

Statements	Reasons
1. $\square ABCD$, $\angle A$ is a rt $\angle$	1. Given
2. $\angle A \cong \angle C$, $\angle B \cong \angle D$	2. Opp. $\angle$s of a $\square$ are $\cong$.
3. $\angle A$ is supp. to $\angle D$	3. Consec. $\angle$s of a $\square$ are supp.
4. $m\angle A = m\angle C$, $m\angle B = m\angle D$	4. Def. of $\cong$ $\angle$s
5. $m\angle A = 90°$	5. Def. of rt $\angle$
6. $m\angle A + m\angle D = 180°$	6. Def. of supp. $\angle$s
7. $m\angle D = 90°$	7. Subt. Prop of = (6 − 5)
8. $m\angle C = 90°$, $m\angle B = 90°$	8. Subst. Prop. of = (5, 7 → 4)
9. $\angle C$, $\angle B$, $\angle D$ are rt $\angle$s	9. Def. of rt $\angle$
10. ABCD is a rectangle	10. Def of Rectangle

Theorem: **If two consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus.**



Given: $\square ABCD$ and $\overline{AB} \cong \overline{AD}$

Prove: ABCD is a rhombus

Statements	Reasons
1. $\square ABCD$, $\overline{AB} \cong \overline{AD}$	1. Given
2. $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$	2. Opp. sides of a $\square$ are $\cong$
3. $\overline{AB} \cong \overline{AD} \cong \overline{DC} \cong \overline{BC}$	3. Trans. Prop. of $\cong$
4. ABCD is a rhombus	4. Def. of rhombus